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Sensitivity Analysis for Discharge Coefficients of Piano Key Weirs

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ABSTRACT

Piano Key Weirs are relatively new weir structures that can be used as a flood release structure at the top of a dam or as a regular weir structure within a river system. The benefit of a PKW is an increased discharge capacity and, hence, lower free surface heads at the upstream reservoir. During the last few years, several experimental and numerical research investigations were carried out to analyze discharge coefficients for various PKW types. Therefore, numerical 3D CFD codes and small-scaled experimental models were used. To identify discharge coefficients, the Poleni formula can be used. By measuring the total discharge and the weir head, discharge coefficients can be calculated. However, results will be sensitive due to the weir head because basically the total energy head includes the velocity head. Hence, the position of the measuring point for the weir head and the associated velocity head must be selected carefully. To show the variation of results for discharge coefficients due to the measurement point and measurement accuracy, a sensitivity analysis for PKW discharge coefficients is given. Measurement data were collected from experimental models at Lübeck University of Applied Sciences' Water Research Laboratory.

Keywords: PKW, Piano Key Weir, discharge coefficients, sensitivity analysis, weir head, velocity head

1. INTRODUCTION AND STATE OF THE ART

Blanc and Lempérière (2001) and Lempérière and Ouamane (2003) developed a general Piano Key Weir design, which is a nonlinear weir type. PKWs can be used as in-channel structures or for top-of-dam spillway control structure to increase discharge capacity (Oertel and Tullis 2014). During the last decades, a significant amount of research investigations were carried out to identify PKW's discharge coefficients (e.g. Anderson 2011, Machiels et al. 2011 and many more).

Generally, PKWs are classified into four main geometric types (Type A, B, C, and D). A Type A PKW features symmetrical keys relative to a transverse centerline axis. Type B has cantilevered apexes on the upstream end and vertical apex walls on the downstream end. Type C is the opposite of Type B. A rectangular labyrinth weir with vertical apex walls is presented by Type D. Primary (crest length, head) and secondary (ratio of inlet and outlet key width and height, overhang length, height of parapet walls) parameters with significant influence on PKW Type A discharge capacity were identified by Ribeiro et al. (2012). Kabiri-Samani and Javaheri (2012) determined discharge coefficients for free and submerged flow over PKWs. Dabbling and Tullis (2012) evaluated and compared submerged head-discharge relationships for Type A PKWs and labyrinth weirs. General flow characteristics over a PKW were investigated by Machiels et al. (2011) within a scaled experimental model. Anderson (2011) and Anderson and Tullis (2012) investigated several PKW geometries and discharge efficiencies for in-channel and reservoir-approach flow conditions. Erpicum et al. (2014) analyzed geometric parameter influence on PKW performances and mentioned e.g. the weir height's primary importance. Experimentally determined discharge coefficients for several PKW types were compared with those from a numerical 3D VOF model by Oertel and Tullis (2014); a good agreement and applicability of the VOF code was found. Additional variations of PKW Type A geometries with

semi-circle and triangle expansions were analyzed by Oertel (2015). In summary, PKW (2011) and PKW (2013) gives an overview about PKW research investigations.

Generally, discharge coefficients C_d for a PKW are in the focus of interest. Figure 1 gives example results for various PKW discharge coefficients (experimental and numerical models) from Anderson and Tullis (2012) and Oertel (2015). It can be shown that the basic development of C_d follows a comparable trend: for small discharges ($H_T P_0 < 0.07$ to 0.11) an increase of C_d can be found; for increasing discharges (after reaching the maximum C_d value, $H_T P_0^{-1} > 0.07$ to 0.11) a continuous decrease of C_d can be observed.

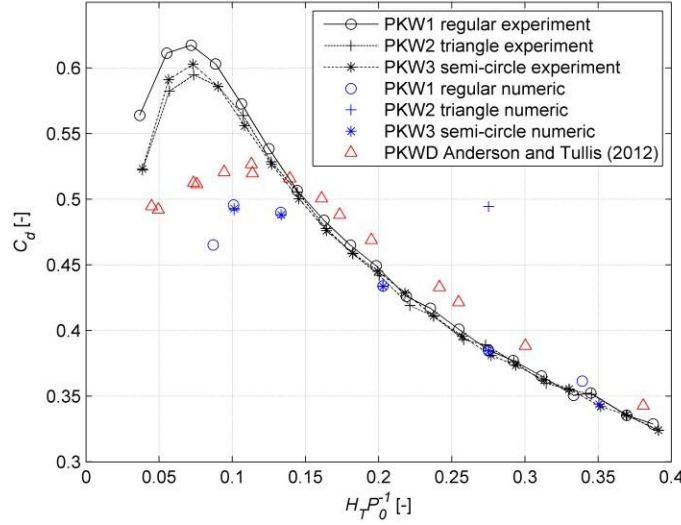


Figure 1. Exemplary discharge coefficients from experimental and numerical models (Oertel 2015).

Using numerical or experimental models, discharge coefficients can be calculated by measuring the upstream water surface level h_T (above weir crest) and calculating the associated velocity head v_T averaged over total flow depth ($h_T + P$, where P is the total weir height for in-channel approach). Figure 2 gives a schematic plot. Subsequently, C_d values can be determined by Poleni formula:

$$Q = \frac{2}{3} C_d L (2g)^{0.5} H_T^{1.5} = C' L H_T^{1.5} \quad (1)$$

where Q = discharge, C_d = dimensionless discharge coefficient, C' = coefficient including $2/3$ and C_d and $(2g)^{0.5}$, L = total centerline crest length, g = acceleration due to gravity, H_T = total upstream energy head including velocity head = $h_T + v_T^2/(2g)$

Aigner (2008) additionally defines discharge coefficient C_d for regular weirs as a function of various influencing parameters; for PKWs, $C_{d,0}$ to $C_{d,3}$ might be of interest:

$$C_d = C_{d,0} \cdot C_{d,1} \cdot C_{d,2} \cdot C_{d,3} \cdot C_{d,4} \cdot C_{d,5} \cdot C_{d,6} \quad (2)$$

where $C_{d,0}$ = basic coefficient (Euler equation), $C_{d,1}$ = inflow velocity coefficient (kinetic energy), $C_{d,2}$ = constriction loss coefficient (head losses), $C_{d,3}$ = jet shape coefficient (velocity distribution), $C_{d,4}$ = backwater coefficient, $C_{d,5}$ = inclined inflow coefficient, and $C_{d,6}$ = pile coefficient.

Since the upstream velocity profile, and hence the water surface level (WSL), will not be uniform, the distance of measurement x_m (ultrasonic sensors for h_T) from the PKW's centerline will influence results for discharge coefficients. Also, the accuracy of measurement techniques might influence investigation results (ultrasonic probes

and magnetic inductive flow meters). Consequently, this paper deals with influencing parameters for discharge coefficient results and their accuracy.

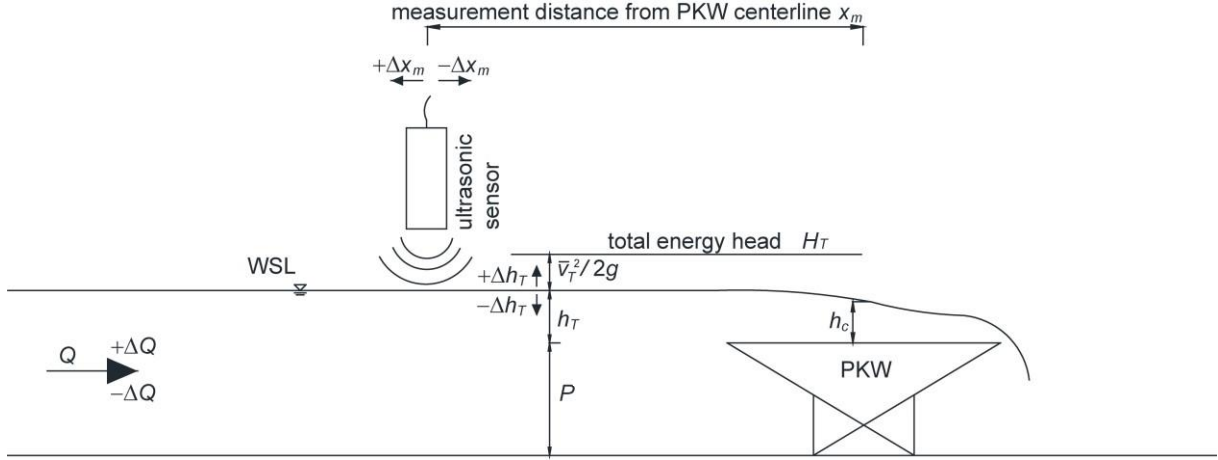


Figure 2. Schematic plot for total energy head (inlet key section, in-channel approach).

2. EXPERIMENTAL MODEL

2.1. Scaled Physical Model and Measurement Technique

To analyze discharge coefficients for PKWs and their accuracy, an experimental model was built up at Lübeck University of Applied Sciences' Water Research Laboratory. The in-channel PKW was fabricated using 5 mm thick acrylic sheeting ($PT_s = 39.4$). The flattop weir crest was machined using a CNC (computerized numerical control) mill to ensure an accurate PKW structure. The as-built dimensions presented in Table 1 were used for all analyses. Parameters are given in Figure 3. The in-channel PKW experiments were conducted in a 0.80 m wide by 0.80 m deep by 10.0 m long rectangular tilting flume. A schematic plot of the experimental model is presented in Figure 4. Flow was supplied into the test flume via piping containing calibrated magnetic inductive flow meters for flow measurement (MID, manufacturer: Krohne, model: Optiflux 2000, accurate to ± 0.1 l/s). An ultrasonic sensor upstream the PKW was used to determine h_T (USS, manufacturer: General Acoustics, model: USS60350, accurate to ± 1 mm). An automatic positioning system performs a predefined measurement program in the x -axis (main flow direction) with approximately 250 measurement points on a 1 cm grid (manufacturer: isel, model: step motor, accurate to ≤ 1 mm). Mean flow velocities were calculated analytically by continuity equation. The total upstream energy head finally is $H_T = h_T + v_T^2/2g$. Using the experimental H_T and Q data, C_d coefficients were calculated using Eq. (1).

Scale effects for PKWs are rarely discussed within the literature (e.g. Machiels et al 2011, Erpicum et al. 2014). Limits for water depths to upscale laboratory-scale results can be found to be approx. 3 cm (see e. g. USDI 1980 or Ribeiro et al. 2012). Since not all experimental model runs were conducted with minimum overtopping water depths of 3 cm, some scale effects for small discharges can be expected due to viscosity or surface tension but without significant influence due to the paper's outcome.

Table 1. Summary of laboratory-scale PKW model dimensions.

Test Weir Dimensions [mm]			
P	196.9	L	4667.2
W	796.0	B_b	230.7

3. RESULTS AND DISCUSSION

3.1. Water Surface Elevation (WSL)

Water surface elevations/levels (WSL) for all investigated discharges are produced by averaging measurements over measuring time (see Section 2.2). All resulting WSL are given in Figure 5. For increasing discharges, a progressive change of the water surface profile directly above the PKW, along with an accelerated flow, can be observed.

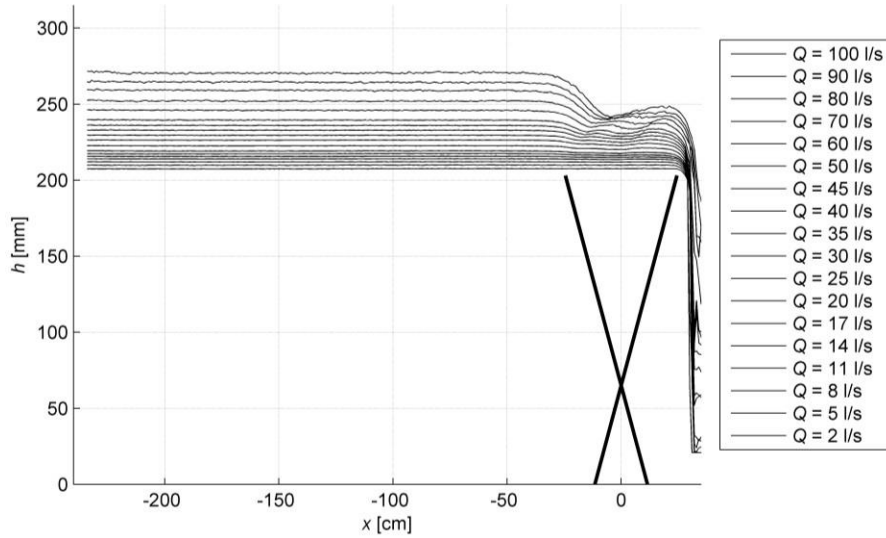


Figure 5. WSL for all investigated discharges at flume's centerline (inlet key section), exaggerated plot.

3.2. Measurement Distance x_m

Model runs are analyzed due to their selected distance of the measurement point, located at a defined distance x_m upstream of the PKW's centerline (see Figure 2). Bollrich (2007) mentioned a measurement distance for overflowed hydraulic structures of $x_m = 3 \times h_T$ to $4 \times h_T$. Since the flow depth above the weir is variable due to changing discharges, the measurement point will change.

Figure 6 presents results for various selected measurement distances between $0.5 \times P$ and $3 \times P$ for channelized PKW applications. Since P is a fixed value, the measurement distance will not change with increasing flow depths above the weir for the present investigation. It can be shown that an absolute distance of $2.0 \times P \leq x_m \leq 3.0 \times P$ will be acceptable for result analyses since discharge coefficients can be reproduced with high accuracy (Figure 6 right).

3.3. Measurement Point x_m vs. Averaged Measurement Area $x_{m,1,2}$

Within an experimental model, small standing waves may occur as a result of small flume lengths or of time-averaged flow depth values. To describe the influence of possible standing waves, C_d values will be calculated on the one hand by measured flow depths at one single location x_m and in contrast by averaged flow depths along a defined measurement area $x_{m,1,2}$. Figure 7 shows results for both approaches. It can be noticed that no major difference in C_d values can be observed, and, hence, no standing waves occur for investigated discharges. Consequently, only the absolute measurement distance x_m is of interest for C_d calculation.

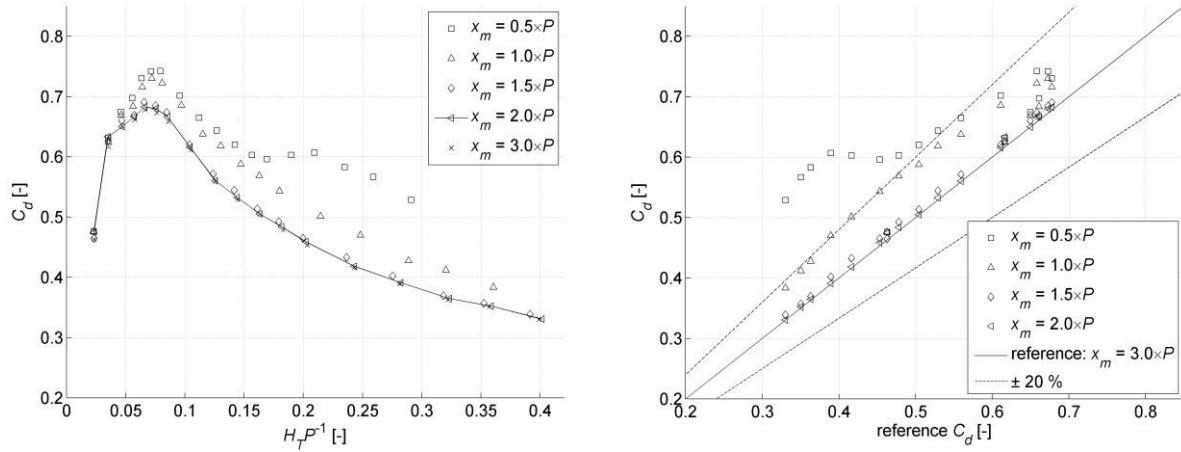


Figure 6. Influence of measurement distance x_m on discharge coefficients C_d , left: absolute values, right: direct comparison.

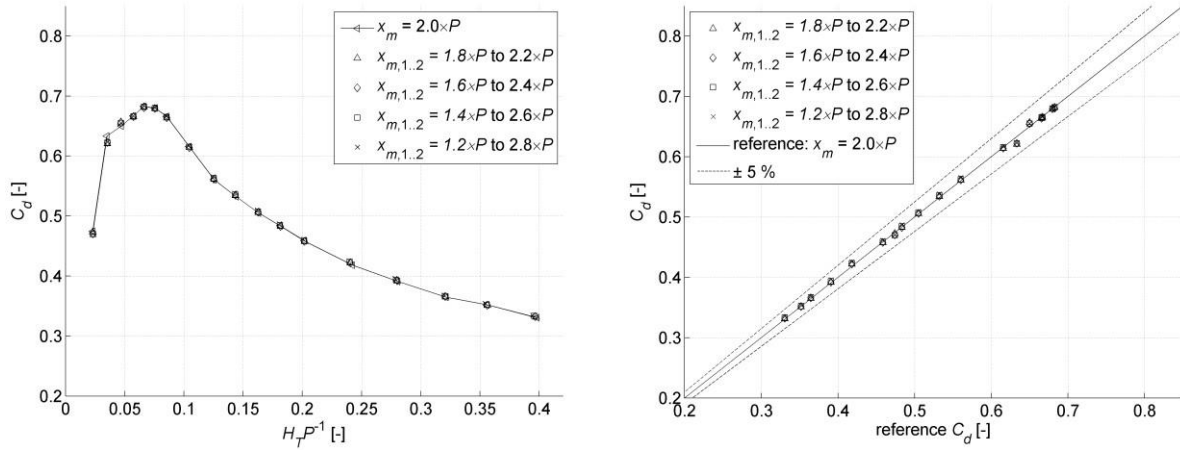


Figure 7. Influence of measurement point x_m vs. averaged measuring area $x_{m,1..2}$ on discharge coefficients C_d , left: absolute values, right: direct comparison.

3.4. Measurement Accuracy of h_T and Q

Ultrasonic probes and MIDs are accurate measurement devices to measure flow depths and discharges within experimental models. The accuracy of used ultrasonic probes can be assumed to be better than 1 mm; for MIDs 0.1l/s. However, results also depend on calibration and offset data. Analogous sensors transfer a defined voltage as output to an AD converter. This converter creates digital values for measurement software products. Hence, a measured voltage represents an exactly associated flow depth or discharge. Therefore, a calibration is necessary to identify the measurement boundary areas.

The ultrasonic probes used provide a voltage range of 0 to 10 V and a measurement distance is 60 to 350 mm. An offset data set allows an identification of the initial flume bed, since the probe is installed somewhere within its measurement range. This may generate some measurement errors or uncertainties.

For results analysis, the original data set will be manipulated. Therefore, the measured flow depths will be increased by $\Delta h_T = 0.5$ or 1.0 mm and decreased by $\Delta h_T = -0.5$ or -1.0 mm. With these new flow depths, new discharge coefficients C_d will be calculated.

Figure 8 gives results for an original data set at $x_m = 2 \times h_T$ as well as for four Δh_T manipulated results. It can be shown that especially for small discharges, (larger C_d values, $H_T P^{-1} < 0.2$) the influence is significant. Already for very small changes in measured flow depths discharge coefficients will be influenced majorly (perhaps partly due to scale effects, which must be quantitatively analyzed in further studies).

Another data set will be manipulated due to measured discharges. Here, the discharge will be increased by 1% or 5% and decreased by -1% or -5%. Plots will be generated using these manipulated values. Results for discharge coefficients with manipulated discharges can be found in Figure 9. Discharge coefficients are calculated using Eq. (1). Consequently, C_d directly depends on Q , and, hence, the percentage accuracy of the discharge measurement device directly leads to the accuracy of discharge coefficient results. Generally, the influence of h_T on resulting discharge coefficients will be increased for increasing discharges (the denominator becomes more important). Thus, the effect of manipulated discharges is larger for small discharges ($H_T P^{-1} < 0.1$).

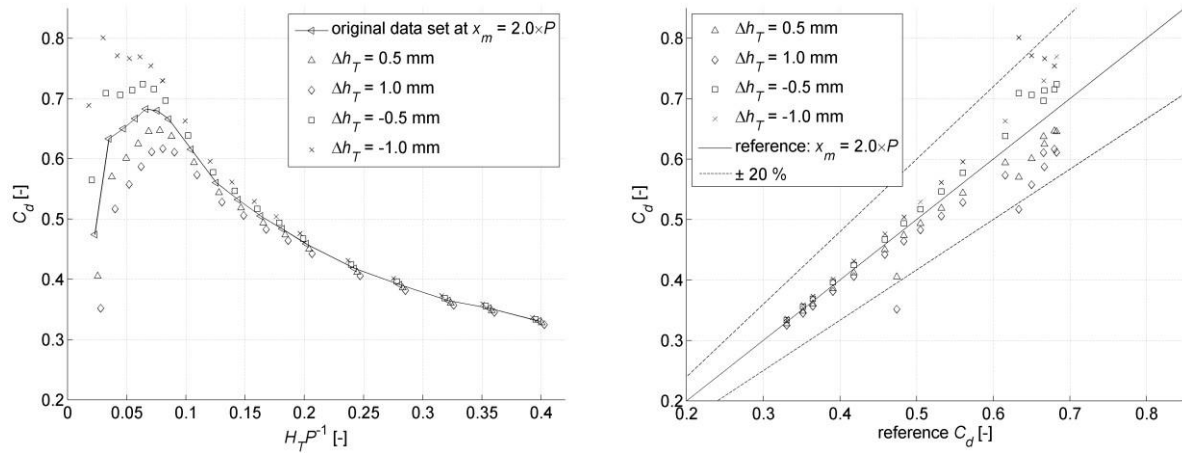


Figure 8. Influence of measurement accuracy h_T on discharge coefficients C_d , left: absolute values, right: direct comparison.

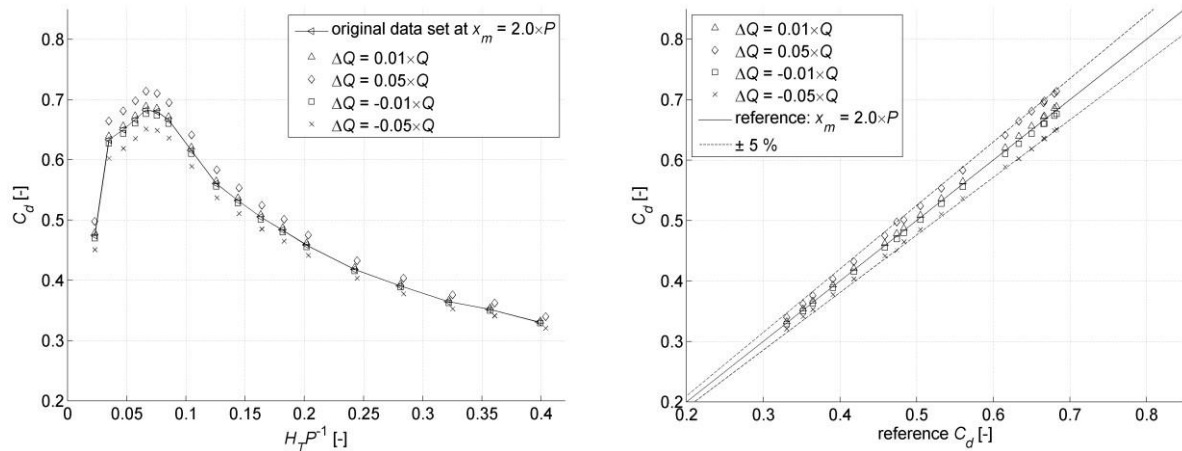


Figure 9. Influence of measurement accuracy Q on discharge coefficients C_d , left: absolute values, right: direct comparison.

3.5. Velocity Head

The literature definitively suggests the inclusion of velocity head into the total energy head for discharge coefficient calculations. To confirm this statement, C_d values will be calculated with and without included velocity heads. Figure 10 clearly shows the difference. Discharge coefficients will be overestimated by up to 20% for nearly all discharges ($H_T P^{-1} > 0.05$). Only for very small flow depths above the PKW ($H_T P^{-1} < 0.03$) are the influence of the velocity head negligible.

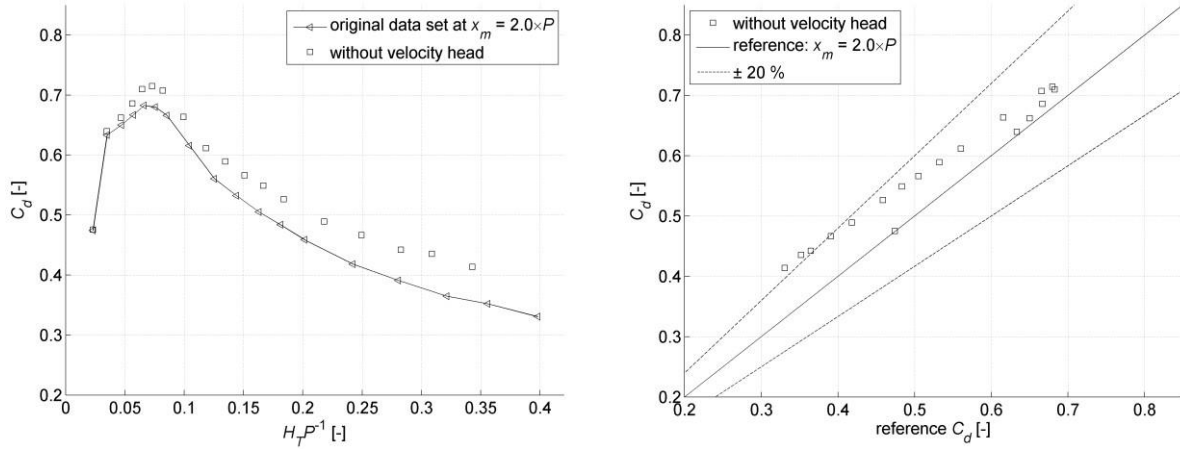


Figure 9. Influence of velocity head $v_T^2 (2g)^{-1}$ on discharge coefficients C_d , left: absolute values, right: direct comparison.

4. SUMMARY AND CONCLUSIONS

In the present paper, a sensitivity analysis for Piano Key Weir discharge coefficients was performed for a selected experimental PKW model at Lübeck University of Applied Sciences' Water Research Laboratory. Five influencing parameters were successively investigated, and their influence on calculated discharge coefficients was quantified for centerline axis (inlet key section).

Main results will be summarized within the following itemization:

- The measurement distance x_m majorly influences calculated discharge coefficients C_d for small x_m values. It is suggested to use values of $x_m = 2 \times P$ for measurement purposes for channelized PKW applications.
- Since no major standing waves occur upstream the PKW within the experimental model, an averaged measurement area will lead to no significant change of C_d values. Hence, a single measurement point at $x_m = 2 \times P$ can be used for accurate results.
- The accuracy of ultrasonic probes majorly influences C_d results. Also, less than 1 mm deviation will increase or decrease discharge coefficients significantly, especially for small discharges.
- The accuracy of discharge meters (MID) has less influence on C_d values since the discharge directly influences results within the Poleni approach. Consequently, the accuracy of calculated C_d values directly comes along with the accuracy of MID measurements.
- Neglecting the velocity head during C_d calculation will generate a difference of up to 20%. Generally, when publishing research works on discharge coefficients, it is essential to mention whether velocity heads were included or not to make own results comparable with values from literature.

The present paper and its sensitivity analysis should provide a guideline for measurement purposes on experimental PKW models within hydraulic laboratories to guarantee comparable results by various researchers and authors.

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